HELPING CHILDREN MASTERY THE BASIC FACTS

Chapter 3

Basic facts for addition and multiplication refer to combinations in which both addends or both factors are less than 10. Subtraction facts can and should be related to the corresponding addition facts, although this is not always done well in traditional curricula. Division facts are likewise closely related to multiplication facts. In contrast with subtraction facts, the multiplication-division connection is generally made quite well. Fluency with the basic facts is developed through a strong understanding of the four operations and an emphasis on conceptual strategies for retrieving the facts.

Mastery of a basic fact means that a child can give a quick response (in about 3 seconds) without resorting to nonefficient means, such as counting. All children are able to master the basic facts—including children with learning disabilities. They simply need to construct efficient mental tools that will help them, which is what this chapter is about.

Big Ideas

1. Number relationships provide the foundation for strategies that help students remember basic facts. For example, knowing how numbers are related to 9 and 10 helps students master facts such as 3 + 6 (think of a ten-frame) and 8 + 6 (since 8 is 2 away from 10, take 2 from 6 to make 10 + 4 + 14).

2. "Think addition" is the most powerful way to think of subtraction facts. Rather than 11 "take away 6," which requires a lot of counting, students can "think 6 and what makes 12." They might add up to 12 or they may think double 6 is 12 so it must be 11.

3. All of the facts are conceptually related. You can figure out new or unknown facts from those you already know. For example, 6 + 8 can be thought of as five 8s (40) and one more 8. It might also be three 8s doubled.

From Concepts and Strategies to Fact Mastery

- Every teacher of grades 4 to 10 knows students who are still counting on their fingers, making marks in the margins to count on, or simply guessing at answers. These students have certainly been given more than adequate opportunity to drill their facts in years past. They have not mastered their facts because they have not developed efficient methods of producing a fact answer. Drill of inefficient methods does not produce mastery.

Fortunately, we know quite a bit about helping children develop fact mastery, and it has little to do with quantity of drill or drill techniques. Three components or steps to this end can be identified:

1. Help children develop a strong understanding of number relationships of the operations.
2. Develop efficient strategies for fact retrieval through practice.
3. Then provide drill in the use and selection of those strategies once they have been developed.

The Role of Number and Operation Concepts

Number relationships play a significant role in fact mastery. The 8 + 6 example in the first big idea requires the relationship between 8 and 10 (8 is 2 away from 10), the part-part-whole knowledge of 6 (2 and 4 makes 6), and the fact that 10 and 4 is 14. For 6 + 7, it is efficient to think "6 times 7 and 7 more." For many children, the efficiency of this approach is lost because they need to count on 7 to get from 35 to 42. With an extension of the number relationships just noted, it is possible to think "35 and 3 more is 40, and 2 more is 42." Every relationship discussed in Chapter 2 can contribute to fact mastery.

The meanings of the operations also play a role in the construction of efficient strategies. The ability to relate 6 + 7 to "5 times 7 and 7 more" is based on an understanding of the meanings of the first and second factors. To relate 13 + 7 to 7 + 7 and what makes 13 requires an understanding of how addition and subtraction are related. The commutative or "turnaround" properties for addition and multiplication reduce the number of addition and multiplication fact from 100 each to 55 each.

Teachers in the upper grades with students who have not mastered basic facts will do well to investigate what command of number relationships and operations the students have. Without these relationships and concepts, the strategies discussed throughout this chapter will be difficult.

Development of Efficient Strategies

An efficient strategy is one that can be done mentally and quickly. The emphasis is on efficient. Counting is not efficient. It drill is undertaken when counting is the only strategy available, all you get is faster counting.

What is a Strategy?

We have already seen some efficient strategies: the use of building up through 10 in adding 8 + 6 and the use of the related fact 5 + 7 to help with 6 + 7.

Stop! Consider for a moment how you think about 6 + 6. What about 9 + 6?

You may think that you just "know" these. What is more likely is that you used some ideas similar to double six (for 6 + 6) and 10 and 4 more (for 9 + 5). Your response may be so automatic by now that you are not reflecting on the use of these
relationships or ideas. That is one of the features of efficient mental processes—they become automatic with use.

Many students have learned basic facts without being taught efficient strategies. They develop or learn many of these methods in spite of the drill they may have endured. The trouble is that far too many students do not develop strategies without instruction and far too many students in middle school continue to count on their fingers. The challenge for teachers is to devise lessons in which all students will develop strategies that are useful. A strategy is most useful to students when it is theirs, built on and connected to concepts and relationships they already own.

For your students to develop effective strategies, you yourself need to have a command of as many good strategies as possible, even if you have never used them. This will help you recognize your students' invented strategies and capitalize on those ideas.

Two Approaches to Fact Strategies

You need to plan lessons or short activities in which specific strategies are likely to be developed. There are two basic types of lessons suggested for this purpose. The first is to use simple story problems designed in such a manner that students are most likely to develop a strategy as they solve it. In the discussion of these solution methods, you can focus attention on the methods that are most useful. You can have all students try the methods others have developed.

A second possible approach is a bit more direct. A lesson may revolve around a special collection of facts for which a particular type of strategy is appropriate. You can discuss how these facts might all be alike in some way, or you might suggest an approach and see if students are able to use it on similar facts.

There is a huge temptation simply to tell students about a strategy and then have them practice it. Though this can be effective for some students, many others will not personally relate to your ideas or may not be ready for them. Continue to discuss strategies invented in your class and plan lessons that encourage strategies.

Drill of Efficient Methods and Strategy Selection

It is inappropriate here to make a distinction between drill and practice. Practice refers to problem-based activities in which students are encouraged to develop (invent, consider, try—but not master) flexible and useful strategies that are meaningful. The types of lessons just described can be thought of as practice lessons. Whether from story problems or from consideration of a collection of similar facts, students are wrestling with the development of strategies that they can use themselves.

Drill refers to repetitive problem-based activity. Drill activity is appropriate for students who have a strategy that they understand, like, and know how to use but have not yet become facile with it. Drill with an in-place strategy focuses students' attention on that strategy and helps to make it more automatic.

Drill plays a significant role in fact mastery, and the use of old-fashioned methods such as flash cards and fact games can be effective if used wisely.

Avoid Premature Drill

It is critical that you do not introduce drill too soon. Suppose that a student does not know the 4 x 6 fact and has no way to deal with it other than to begin to skip count, 6, 12, and then continue to count fingers by ones to get to 24. This is an inef-

cient method. Premature drill introduces no new information and encourages no new connections. It is merely a waste of time and a frustration to the child.

As you read through this chapter, you may feel that the strategies for some facts, especially the harder multiplication facts, do not seem to be efficient at all. However, as long as the strategy is completely mental and does not rely on a model, picture, or tedious counting, repeated use of the strategy will almost certainly render it automatic.

The strategy provides a mental path from fact to answer. Soon the fact and answer are "connected" as the strategy becomes almost unconscious.

The discussion in this chapter focuses one strategy at a time. It is not at all unreasonable for students to be engaged in drill activities with one strategy before they have developed (via practice or problem-based activity) strategies for other facts.

Many of the activities suggested in the chapter are simple drills—flash cards, matching games, dice, or spinner activities—in which the objective is quick response. Do not misinterpret these activities that are clearly drills as the way to introduce or develop strategies. Drill should only be used when an efficient strategy is in place.

Practice Strategy Selection or Strategy Retrieval

Strategy selection or strategy retrieval is the process of deciding what strategy is appropriate for a particular fact. If you don't think to use a strategy, you probably won't.

Many teachers who have tried teaching fact strategies report that the method works well while the students are focused on whatever strategy they are working on. They acknowledge that students can learn and use strategies. But, when they continue, the facts are all mixed up or the student is not in "fact practice" mode, old counting habits return.

For example, suppose that your students have been practicing the near-doubles facts for addition: Use the better-known double, 7 + 7, to derive the unknown 8 + 7. Students become quite skilled at doubling the smaller number and adding 1. All of the facts they are practicing are selected to fit this model. On other days, they have learned and practiced other strategies. Later, on a worksheet or in a mental math exercise, the children are presented with a mixture of facts. In a single exercise, a child might see

[Table]

<table>
<thead>
<tr>
<th>7</th>
<th>4</th>
<th>2</th>
<th>8</th>
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<tbody>
<tr>
<td>+6</td>
<td>+9</td>
<td>+6</td>
<td>+5</td>
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There is no mind-set or reminder to use different processes for each. Especially if the children have previously been habituated to counting to get answers, they will very likely revert to counting and ignore the efficient methods that were the focus of recent drills. When they were drilling the strategy, there was no need to decide what strategy might be useful. All of the facts in the near-doubles practice were near-doubles, and the strategy worked. Later, however, there is no one to suggest the strategy.

A simple activity that is useful is to prepare a list of facts selected from two or more strategies and then, once fact at a time, ask students to name a strategy that would work for that fact. They should explain why they picked the strategy and demonstrate its use. This type of activity turns the attention to the features of a fact that lend it to this or that strategy.

Overview of the Approach

For each particular strategy, from development to eventual drill when the strategy is well understood, the general approach for instruction is very similar.
**Make Strategies Explicit in the Classroom**

As has been discussed, your students will develop strategies as they solve word problems or as they investigate a category of facts you present. When a student suggests a new strategy, be certain that everyone else in the room understands how it is used. Suppose that Helen explains how she figured out $3 \times 7$ by starting with double $7 (14)$ and then adding $7$ more. She knew that $6$ more onto $14$ is $20$ and one more is $21$. You can ask another student to explain what Helen just shared. This requires students to attend to ideas that come from their classmates. Now explore with the class to see what other facts would work with Helen's strategy. This discussion may go in a variety of directions. Some may notice that all of the facts with a $3$ in them will work. Others may say that you can always add one more set on if you know the smaller fact. For example, for $6 \times 8$ you can start with $5 \times 8$ and add $8$.

Don't expect to have a strategy introduced and understood with just one word problem or one exposure such as this. Try on several successive days problems in which the same type of strategy might be used. Students need lots of opportunities to make a strategy their own. Many students will simply not be ready to use an idea the first few days, and then all of a sudden something will click and a useful idea will be theirs.

It is a good idea to write new strategies on the board or make a poster of strategies students develop. Give the strategies names that make sense. (Double and add one more set. Helen’s idea. Use with 2s. Include an example.)

No student should be forced to adopt someone else’s strategy, but every student should be required to understand strategies that are brought to the discussion.

**Drill Established Strategies**

When you are comfortable that children are able to use a strategy without recourse to skip counting and that they are beginning to use it mentally, it is time to drill it. You might have as many as ten different activities for each strategy or group of facts. File folder or boxed activities can be used by students individually, in pairs, or even in small groups. With a large number of activities, students can work on strategies they understand and on the facts that they need the most.

Flash cards are among the most useful approaches to fact strategy practice. For each strategy, make several sets of flash cards using all of the facts that fit that strategy. On the cards, you can label the strategy or use drawings or cues to remind the children of the strategy. Examples appear throughout the chapter.

Other activities involve the use of special dice made from wooden cubes or foam rubber, teacher-made spinners, matching activities in which a helping fact or a relationship is matched with the new fact being learned, and games of all sorts. A game or drill suggested for one strategy can usually be adapted to another.

**Individualise**

To some extent, you want to individualise drills in such a way that students are using their preferred strategy in the drills. This is not as difficult as it may seem at first.

Different students will likely invent or adopt different strategies for the same collection of facts. For example, there are several methods or strategies that use $10$ when adding $8$ or $9$. Therefore, a drill that includes all of the addition facts with an $8$ or a $9$ can accommodate any child who has a strategy for that collection. Two children can be playing a spinner drill game, each using different strategies.

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**Assessment Note**

It is imperative that you listen to your students. Keep track of what strategies your students are using. This will help you occasionally create examples that get all the students to benefit from the same drill. This will not only help your students have yet to develop an efficient strategy for each fact, but it also helps them to see that more than one way of doing things exists and then compare and contrast the different ways of doing things. This simple fact test with facts that have not yet been mastered will help you determine if your students need to be given a diagnostic test, a simple fact test with facts that have yet to be mastered, or if you want them to first answer only those facts that are not mastered. Then they should go back and attempt the facts that are mastered until they can use the strategies and apply them to all the facts.

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**Practice Strategy Selection**

After students have worked on two or three strategies, strategy selection drills are very important. These can be conducted quickly with the full class or a group, or independent games and activities can be prepared. Examples are described toward the end of the chapter.

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**Strategies for Addition Facts**

- **Addition facts**—the sums through $18$—are generally considered mastery items in the second-grade curriculum. However, very few third-grade teachers will ever see a new class that has mastered all of these facts, and many teachers of fifth grade and higher have students who have not mastered these facts.

  For teachers in grades $3-5$, the following ideas are important:

  - All of the addition facts can be connected to a handful of very important number relationships. For students who have not mastered addition facts, more time will be saved by some attention to those relationships than with more drill.
  - It is almost never the case that a student will know a subtraction fact without also knowing the corresponding addition fact. That is, if a student knows $12 - 8$, it is almost certain that he or she also knows $8 + 4$. Therefore, mastery of addition facts should be seen as a prerequisite to subtraction facts.
  - Diagnosis of what facts individual students have mastered will help you plan a method of helping your students. Time spent in this manner will save time in the long run.

  The following sections are intended only to provide a brief look at the possibilities of strategies for addition facts. With this information you can plan efficient activities to help students master these facts quickly.

**Facts with a Zero, One, or Two**

As shown in the following table, $51$ of the $100$ addition facts involve a $0$, $1$, or a $2$. 
Students who are missing facts with a zero are likely responding based on a faulty notion that "addition makes bigger." Therefore, they might answer "8" to 7 + 0. These facts do not require any strategy but rather a good understanding of the meaning of zero and addition.

Many students in grades 1 and 2 have been taught to use a "count-on" strategy for facts with addends of 1 or 2 as well as those involving a 3. This is a common strategy found in traditional textbooks. If students are using a counting-on strategy efficiently—and using it only for these small addends—do not try to stop it. However, we strongly suggest that you not encourage the use of counting on for any facts. It is difficult for students to separate counting on for some facts and not for others. Intermediate-grade students who have not yet mastered addition facts are probably counting on for 8 + 5 and other facts for which counting is certainly not efficient.

Rather than counting on, consider focusing on the relationships of one-more than and two-more-than: 8 is 2 more than 6; one more than 3 is 4. Try simple drills in which a die is thrown, a numeral card is turned up, or a ten-frame card is shown and students respond with the number that is one-more-than (or two-more-than) the given number. Students who become proficient with this relationship can then be helped to connect it to the corresponding addition facts. Two drill ideas are shown in Figure 3.1.

It is helpful to point out to students that with only an understanding of 0 and the one- and two-more-than ideas, they know 51 addition facts. This is especially powerful for a fourth or fifth grader who is struggling to learn the facts.

**Doubles and Near-Doubles Facts**

It is well documented that children and adults alike seem to know the doubles facts (both addends alike) better than most other combinations. There are only 10 doubles facts and only seven of these have addends of 3 or greater. However, these seven facts provide a useful anchor for facts often referred to as the near-doubles: facts such as 6 + 7 or 5 + 4 in which the addends are only one apart. The strategy for the near-doubles is to double the smaller addend and add 1. The doubles and near-doubles facts are shown in the table to the right. When the doubles and near-doubles are added on to the 0, 1, and 2 facts, a total of 70 facts are accounted for. To help students with these facts, first check to see if they know the doubles facts. If not, here are two suggestions.

Have students solve simple word problems that focus on a pair of like addends. Alex and Zack each found 7 seashells at the beach. How many did they find all together? Students should solve the problem mentally. Then have several students share their solutions.

The following activity is a useful drill that is easily individualized and requires no preparation on your part.

**ACTIVITY 3.1**

**Calculator Doubles**

Students first make their calculator into a "double maker" by pressing 2 2. Working in pairs, one student says a double fact, for example "double seven." The student with the calculator first presses a 7 and then tries to give the sum (without counting). He or she then presses = to see the correct double on the display.

To make a double maker with some calculators may require pressing 2 3 or using an operation key. (Note that the calculator is also a good tool for practicing the +1 and +2 facts.)

When students know the doubles, they can quickly connect these to the near-doubles. Simple story problems are again a good method for getting the double-plus-one strategy into the classroom. For a story problem involving a near-double, it is highly probable that some students will use the double-plus-one approach. Note that some students may double the larger addend and subtract one. Encourage any strategy that seems helpful to the individual student. However, students with weak number concepts sometimes apply double-plus-one incorrectly by beginning with the larger addend rather than the smaller. For example, they may use double 7 plus 1 for 7 + 6. Therefore, it is a good idea to focus especially on the doubling of the smaller addend.

If you want to discuss this (or any strategy) with the full class, you can use the following approach. Write approximately ten near-doubles facts on the board. Vary which addend is smaller. Have students work independently to write the answers. Then discuss their ideas for "good" (i.e., efficient) methods of answering these facts. As with the use of a story problem, it is almost certain that some students will use a double-plus-one approach. When this happens, focus on that method by having all students try the strategy with other near-doubles facts.

When the strategy is clear to students, drill activities similar to those shown in Figure 3.2 are appropriate.

**Make Ten Facts**

These facts all have at least one addend of 8 or 9. One strategy for these facts is to build onto the 8 or 9 up to 10 and then add on the rest. For 6 + 8, start with 8, then 2 more makes 10, and that leaves 4 more for 14.
Before using this strategy, be sure that students have learned to think of the numbers 11 to 18 as 10 and some more. Many second- and third-grade children have not constructed this relationship.

The next activity is a good way to introduce the make-ten strategy.

**ACTIVITY 3.2**

**Make 10 on the Ten-Frame**

Give students a mat with two ten-frames (see Figure 3.3). Flash cards are placed next to the ten-frames or a fact can be given orally. The students should first model each number in the two ten-frames and then decide on the easiest way to show (without counting) what the total is. The obvious (but not the only) choice is to move counters into the frame showing either 8 or 9. Get students to explain what they did. Focus especially on the idea that 1 (or 2) can be taken from the other number and put with the 9 (or 8) to make 10. Then you have 10 and whatever is left is 8.

Make-ten with two ten-frames.

Provide a lot of time with the make-ten activity. Encourage discussion and exploration of "easy ways" to think about adding two numbers when one of them is 8 or 9. Perhaps discuss why this is not a useful idea for a fact such as 6 + 5 where neither number is near 10.

Note that children will have many other ways of using 10 to add with 8 or 9. For example, with the fact 9 + 5, some will add 10 + 5 and subtract 1. This is a perfectly good strategy, and it uses 10. You may want to give efficient strategies unique names determined by the students and discuss which ones seem especially useful.

With two ten-frames showing another common strategy is to use a row of 5 from each frame to make ten. For the 7 + 9 fact in Figure 3.2, this strategy would first take the two rows of 5 (10) and then add 3 + 4, the leftover parts of each number. Interestingly, this is a popular strategy in Japan. Perhaps more importantly, this idea can be used with any fact in which each addend is 5 or more. There are 25 such facts and they include those typically thought of as difficult to learn.

When children seem to have the make-ten idea or a similar strategy, try the same activity without counters. Use the little ten-frame cards found in the Blackline Masters. Students can set out an 8 (or 9) card on their desk and place other cards beneath it one at a time. Suggest mentally "moving" two dots into the 8-ten-frame. Have students say orally what they are doing. For 8 + 4, they might say, "Take 2 from the 4 and put it with 8 to make 10. Then 10 and 2 left over is 12." The activity can be done independently with the little ten-frame cards.

As has been noted, there is more than one way to efficiently use 10 in a strategy for facts involving 8 or 9. Imagine two or three children who have on the table a small ten-frame card for 9 so that all can see. One at a time, other cards are turned up and the students are to name the total of the two cards. How many different efficient methods involving 10 can you think of that can be accomplished by this simple activity?

**FIGURE 3.3**

Other Strategies and the Last Six Facts

To appreciate the power of strategies for fact learning, consider the following. We have discussed only five ideas or strategies (one- or two-more than, zeros, doubles, near-doubles, and make-ten) yet these ideas have covered 88 of the 100 addition facts! Furthermore, these ideas are not really new but rather the application of important relationships. The 12 remaining facts are really only six facts and their respective turnarounds as shown on the chart.

Before trying to develop any particular strategies for these facts, spend several days with word problems in which these facts are the addends. Listen carefully to the ideas that students use in figuring out the answers.

**Doubles Plus Two, or Two-Apart Facts**

Of the six remaining facts, three have addends that differ by 2: 3 + 5, 4 + 6, and 5 + 7. There are two possible relationships that might be useful here, each depending on knowledge of doubles. Some children find it easy to extend the idea of the near-doubles to double plus 2. For example, 4 + 6 is double 4 and 2 more. A different idea is to take 1 from the larger addend and give it to the smaller. Using this idea, the 5 + 3 fact is transformed into the double 4 fact—double the number in between.

**Make-Ten Extended**

Three of the six facts have 7 as one of the addends. The make-ten strategy is frequently extended to these facts as well. For 7 + 4, the idea is 7 and 3 near 10 and 1 left is 11. You may decide to suggest this idea at the same time that you initially introduce the make-ten strategy.

**Ten-Frame Facts**

If you have been keeping track, five of the remaining six facts have been covered by the discussion so far, with a few being touched by two different thought patterns. Only 6 + 3 has been neglected. The ten-frame model is so valuable in seeing certain number relationships that these ideas cannot be passed by in thinking about facts. The ten-frame helps children learn the combinations that make 10. Ten-frames immediately model all of the facts from 5 + 1 to 5 + 5 and the respective turnarounds. Even 5 + 6, 5 + 7, and 5 + 8 are quickly seen as two facts and some more when depicted with these powerful models (see Figure 3.4).

A good idea might be to group the facts shown in the chart here and practice them using one or two ten-frames as a cue to the thought process.

The next two activities are suggestive of the type of relationships that can be developed.

**FIGURE 3.4**

Ten-Frame Facts.
**ACTIVITY 3.3**

**A Plus-Five Machine**

Use the calculator to practice adding five. Enter \(+5\). Next enter any number and say the sum of that number plus 5 before pressing \(=\). Continue with other numbers. (The \(+5\) need not be repeated.) If a ten-frame is present, the potential for strengthening the 5 and 10 relationships is heightened.

Obviously, the calculator can be made into a machine for adding any number and is a powerful drill device.

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**ACTIVITY 3.4**

**Say the Ten Fact**

Hold up a ten-frame card and have students say the "ten fact." For a card with 7 dots, the response is "seven and three is ten." Later, with a blank ten-frame drawn on the board, say a number less than 10. Students start with that number and complete the "ten fact." If you say, "four," they say, "four plus six is ten." Use the same activities in independent or small group modes.

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**Strategies for Subtraction Facts**

- Subtraction facts prove to be more difficult than addition. This is especially true when children have been taught subtraction through a "count-count-count" approach; for 13 - 5, count 13, count off 5, count what's left. There is little evidence that anyone who has mastered subtraction facts has found this approach helpful. Unfortunately, many sixth, seventh, and eighth graders are still counting.

**Subtraction as Think-Addition**

In figure 3.5 subtraction is modeled in such a way that students are encouraged to think, "What goes with this part to make the total?" When done in this think-addition manner, the child uses known addition facts to produce the unknown quantity or part. If this important relationship between parts and wholes—between addition and subtraction—can be made, subtraction facts will be much easier. When children see 9 - 4, you want them to think spontaneously, "Four and what makes nine?" By contrast, observe a third-grade child who struggles with this fact. The idea of thinking addition never occurs. Instead, the child will begin to count either back from 9 or up from 4. The value of think-addition cannot be overstated.

Word problems that promote think-addition are those that sound like addition but have a missing addend. Consider this problem: Janice had 5 fish in her aquarium. Grandma gave her some more fish. Then she had 12 fish. How many fish did Grandma give Janice? Notice that the action is join and, thus, suggests addition. There is a high probability that students will think 5 and how many more makes 12. In the discussion in which you use problems such as this, your task is to connect this thought process with the subtraction fact, 12 - 5.

**Subtraction Facts with Sums to 10**

Think-addition is most immediately applicable to subtraction facts with sums of 10 or less. Sixty-four of the 100 subtraction facts fall into this category.

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**Assessment Note**

Using a combination of both effectively, addition facts must be mastered. Since most students have not yet mastered their addition facts, give them a page of them, and a different page of addition facts. To minimize confusion between the addition and subtraction facts easier for you to verify the facts on the two pages. That is, put the addition fact 5 + 4 in the subtraction fact 5 - 1 on the addition page as 9 - 4 on the subtraction page. This should also help be those facts that they know quickly without thinking or counting. Explain that you only want to find out what they can. You can help them with those facts they have yet to master.

Many students have not mastered all or nearly all of their addition facts to the point where they may have to begin. The paired addition and subtraction facts approach encourage students to use addition facts they already know but subtraction facts are not known, thus providing help in developing subtraction facts.

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**The 36 “Hard” Subtraction Facts: Sums Greater Than 10**

Before reading further, look at the three subtraction facts shown here and try to reflect on what thought process you use to get the answers. Even if you "just know them," think about what a likely process might be.

\[14 - 9 = 5, \quad 12 - 6 = 6, \quad 18 - 6 = 12\]

Many people will use a different strategy for each of these facts. For 14 - 9, it is easy to start with 9 and work up through 10 - 9 and 1 more is 5. For the 12 - 6 fact, it is quite common to hear "double 6," a think-addition approach. For the last fact, 15 - 6, 10 is used again but probably by working backward from 15—a take-away process: Take away 5 to get 10, and 1 more makes 15. We could call these three approaches, respectively, build up through 10, think-addition, and back down through 10. Each of the remaining 36 facts with sums of 11 or more can be learned using one or more of these strategies. Figure 3.6 shows how these facts, in three overlapping groups, correspond to these three strategies. Keep in mind that these are not required strategies. Some students may use a think-addition method for all. Others may have a completely different strategy for some or all of these. The three approaches suggested here are...
FIGURE 3.6  **********
The 34 “Hard” subtraction facts.

![Subtraction Facts Diagram]

Based on ideas already developed: the relationship between addition and subtraction and the power of 10 as a reference point.

Note that the build-up group includes all facts in which the part or subtracted number is either 8 or 9. Examples are 13 – 8 and 15 – 8. In contrast, the back-down-through-ten facts are really take-away and not think-addition. It is useful for facts where the ones digit of the whole is close to the number being subtracted. For example, with 15 – 6, you start with the total of 15 and take of 5. That gets you down to 10. Then take off 1 more to get 9. For 14 – 6, just take off 4 and then take off 2 more to get 8. Here we are working backward with 10 as a “bridge.”

**Extend Think-Addition**

Think-addition remains one of the most powerful ways to think about subtraction facts. When the think-addition concept of subtraction is well developed, many children will use that approach for all subtraction facts. (Notice that virtually everyone uses a think-multiplication approach for division. Why?)

What may be most important is to listen to children’s thinking as they attempt to answer subtraction facts that they have not yet mastered. If they are not using one of the three ideas suggested here, it is a good bet that they are counting—an inefficient method.

The activities that follow are all of the think-addition variety. There is, of course, no reason why these activities could not be used for all of the subtraction facts. They need not be limited to the “hard facts.”

ACTIVITY 3.5  **********

**Missing-Number Cards**

Show children, without explanation, families of numbers with the sum circled as in Figure 3.7(a). Ask why they think the numbers go together and why one number is circled. When this number family idea is fairly well understood, show some families with one number replaced by a question mark, as in Figure 3.7(b), and ask what number is missing. When students understand this activity, explain that you have made some missing-number cards based on this idea. Each card has two of three numbers that go together in the same way. Sometimes the circled number is missing (the sum), and sometimes one of the other numbers is missing (a part). The object is to name the missing number.

**ACTIVITY 3.6  **********

**Missing-Number Worksheets**

Make copies of the blank form found in the Blackline Masters to make a wide variety of drill exercises. In a row of 13 “cards,” put all of the combinations from two families with different numbers missing, some parts and some wholes. Put blanks in different positions. An example is shown in Figure 3.8. After filling in numbers, run the sheet off, and have students fill in the missing numbers. Another idea is to group facts from one strategy or number relation or perhaps mix facts from two strategies on one page. Have students write an
addition fact and a subtraction fact to go with each missing-number card. This is an important step because many children are able to give the missing part in a family but do not connect this knowledge with subtraction.

Strategies for Multiplication Facts

Multiplication facts can and should be mastered by relating new facts to existing knowledge.

It is imperative that students completely understand the commutative property (go back and review Figure 2.16, p. 66). For example, 2 × 8 is related to the addition fact double 8. But the same relationship also applies to 8 × 2 that many students think about as 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8. Most of the fact strategies are more obvious with the factors in one order than in the other, but turnaround facts should always be learned together.

Or of the five groups of strategies discussed next, the first four strategies are generally easier and cover 75 of the 100 multiplication facts. You are continually reminded that these strategies are suggestions, not rules, and that the most general approach with children is to have them discuss ways that they can use to think of facts easily.

Doubles

Facts that have 2 as a factor are equivalent to the addition doubles (e.g., 7 + 7) and should already be known by students who know their addition facts. The major problem is to realize that not only is 2 × 7 double 7, but so is 7 × 2. Try word problems where 2 is the number of sets. Later use problems where 2 is the size of the sets.

Make and use flash cards with the related addition fact or word double as a clue (see Figure 3.9).

Fives Facts

This group consists of all facts with 5 as the first or second factor, as shown here.

Practice counting by fives to at least 45. Connect counting by fives with rows of 5 dots (see Figure 3.10). Point out that six rows is a model for 6 × 5, eight rows is 8 × 5, and so on.

ACTIVITY 3.7

Clock Facts

Focus on the minute hand of the clock. When it points to a number, how many minutes after the hour is it? Draw a large clock face, and point to numbers 1 to 9 in random order. Students respond with the minutes after. Now connect this idea to the multiplication facts with 5. Hold up a flash card and then point to the number on the clock corresponding to the other factor. In this way, the fives facts become the “clock facts.” Include the clock idea on flash cards or to make matching activities (see Figures 3.10).

Zeros and Ones

Thirty-six facts have at least one factor that is either 0 or 1. These facts, though apparently easy, sometimes confuse students with “rules” they may have learned for addition. The fact 6 × 0 stays the same, but 6 × 0 is always zero. The 1 × 4 fact is a one-more idea, but 1 × 4 stays the same. The concepts behind these facts can be developed best through story problems. Above all else, avoid rules that sound arbitrary and without reason such as “Any number multiplied by zero is zero.”

Nifty Nines

Facts with a factor of 9 include the largest products but can be among the easiest to learn. The table of nine facts includes some nice patterns that are fun to discover. Two of these patterns are useful for mastering the nines: (1) The tens digit of the product is always
one less than the "other" factor (the one other than 9), and (2) the sum of the two digits in the product is always 9. These two ideas can be used together to get any nine fact quickly. For 7 × 9, 1 less than 7 is 6, and 3 makes 9, so the answer is 63.

Students are not likely to invent this strategy simply by solving word problems involving a factor of 9. Therefore, consider building a lesson around the following task.

**Activity 3.8**

Patterns in the Nines Facts

In column form, write the nines table on the board (9 × 1 = 9, 9 × 2 = 18, ..., 9 × 9 = 81). The task is to find as many patterns as possible in the table. (Do not ask students to think of a strategy.) As you listen to the students work on this task, be sure that somewhere in the class the two patterns necessary for the strategy have been found. After discussing all the patterns, a follow-up task is to ask the patterns to think of a clever way to figure out a nine fact if you didn’t know it. (Note that even for students who know their nines facts, this remains a valid task.)

Once students have invented a strategy for the nines, practice activities such as those shown in Figure 3.11 are appropriate. Also, consider word problems with a factor of 9 and check to see if the strategy is in use.

Warning: Although the nines strategy can be quite successful, it can also cause confusion. Because two separate rules are involved and a conceptual basis is not apparent, students may confuse the two rules or attempt to apply the idea to other facts. It is not, however, a "rule without reason." It is an idea based on a very interesting pattern that exists in the base-ten number system. One of the values of patterns in mathematics is that they help us see seemingly difficult things quite easily. The single-digit pattern illustrates clearly one of the values of pattern and regularity in mathematics.

An alternative strategy for the nines is almost as easy to use. Notice that 9 × 9 is the same as 7 × 10 less one set of 7, or 70 - 7. This can be easily modeled by displaying rows of 10 cubes, with the last one a different color, as in Figure 3.12. For students who can easily subtract 4 from 10, 5 from 50, and so on, this strategy may be preferable.

You might introduce this idea by showing a set of bars such as those in the figure with only the end cube a different color. After explaining that every bar has ten cubes, ask students if they can think of a good way to figure out how many are yellow.

**Helping Facts**

The following chart shows the remaining 25 multiplication facts. It is worth pointing out to students that there are actually only 15 facts remaining to master because 20 of them consist of 10 pairs of turnarounds.

These 25 facts can be learned by relating each to an already known fact or helping fact. For example, 3 × 8 is connected to 2 × 8 (double 8 and 8 more). The 6 × 7 fact can be related to either 5 × 7 (5 sevens and 7 more) or to 3 × 7 (double 3 × 7). The helping fact must be known, and the ability to do the mental addition must also be there. For example, to go from 5 × 7 to 6 × 7, a student must be able to efficiently add 35 and 7.

**Assessment Note**

How to find a helping fact that is useful varies with different facts and sometimes depends on which factor you focus on. Figure 3.13 illustrates models for four overlapping groups of facts and the thought process associated with each.

The double and double again approach is applicable to all facts with 4 as one of the factors. Remember that the idea works when 4 is the second factor as well as when it is the first. For 4 × 6, double 16 is also a difficult fact. Help children with this by noting, for example, that 15 + 15 is 30, 16 + 16 is two more, or 32. Adding 16 + 16 on paper defeats the purpose.

Double and one more set is a way to think of facts with one factor of 5. With an array or a set picture, the double part can be circled, and it is clear that there is one more set. Two facts in this group involve difficult mental additions. If either factor is even, a half then double approach can be used. Select the even factor, and cut it in half. If the smaller fact is known, that product is doubled to get the new fact. For 6 × 7, half of 6 is 3. Three times 7 is 21. Double 21 is 42. For 8 × 7, the double of 28 may be hard, but it remains an effective approach to that traditionally hard fact. (Double 25 is 50 + 2 times 25 is 55 or double 30 is 60, 60 subtract 3 is 57.)

Many children prefer to go to a fact that is "close" and then add one more set to this known fact. For example, think of 6 × 7 as 6 sevens. Five sevens is close: That’s 35. Six sevens is one more seven, or 42. When using 5 × 8 to help with 6 × 8, the set language "6 eights" is very helpful in remembering to add 8 more and not 6 more. Admittedly difficult, this approach is used by many children, and it
becomes the best way to think of one or two particularly difficult facts. "What is seven times eight? Oh, that's 8 sevens or 49 and 7 more—56." The process can become almost automatic.

The relationships between easy and hard facts are fertile ground for good problem-solving tasks. Rather than tell students what helping facts to use and how to use them, select a hard multiplication fact and challenge students to find interesting and useful ways to answer it. This approach is described in the next activity.

**Activity 3.9**

**If You Didn't Know**

Pose the following task to the class: If you didn’t know the answer to $6 \times 8$ (for any fact that you want students to think about), how could you figure it out by using something that you do know? Explain to students that their method should be something that they can do in their head and should not rely on counting. Encourage students to come up with more than one way.

Use a think-pair-share approach in which students discuss their ideas with a partner before they share them with the class.

Go through each of the 20 "hard facts" and see how many of the strategies in Figure 3.13 you can use for each one. Many students will not think in terms of the arrays shown but rather will use a symbolic representation. For example, for $6 \times 8$ they might think of a vertical sum of 6 8s or eight 6s. Try to see how this type of representation works for the ideas in Figure 3.13.

Since arrays are a powerful thinking tool for these strategies, provide students with copies of the ten-by-ten dot array found in the Blackline Masters. A tagboard can be used to outline specific product arrays as shown in Figure 3.14. The lines in the array make counting the dots easier and often suggest the use of the easier five facts as helpers. For example, $7 \times 7$ is $5 \times 7$ plus double $7 \rightarrow 35 + 14$.

Don't forget to use word problems as a vehicle for developing these harder facts. Consider this problem: Correne banded up all of her old crayons into bags of 7 crayons each. She was able to make 8 bags with 3 crayons left over. How many crayons did she have? As students work to answer this question, many of the strategies just discussed are possible. Plus there is the added benefit of the assessment value gained by listening to the methods different students bring to a situation that does not look like a fact drill.

Word problems can also be structured to prompt a strategy: Carlos and Joe kept their baseball cards in albums with 6 cards on each page. Carlos had 4 pages filled, and Joe had 8 pages filled. How many cards did each boy have? (Do you see the halfthen-double strategy?)

It should be clear that arrays and set pictures play a large part in helping students establish multiplication facts and relationships. They can be used to help with multiplication facts, the relationship between multiplication and division, and in the development of computational procedures for multiplication. They provide students with a visual image of strategies. As they understand the strategy, move away from these visuals in an effort to build efficiency.

**Division Facts and "Near Facts"**

**Stop** What thought process do you use to recall facts such as $48 \div 6$ or $36 \div 9$?

If we are trying to think of $36 \div 9$, we tend to think, "Nine times what is thirty-six?" For most, $42 \div 6$ is not a separate fact but is closely tied to $6 \times 7$. (Would it not be wonderful if subtraction were so closely related to addition? It can be!)

An interesting question to ask is, "When children are working on a page of division facts, are they practicing division or multiplication?" There is undoubtedly some value in limited practice of division facts. However, mastery of multiplication facts and connections between multiplication and division are the key elements of division fact mastery. Word problems continue to be a key vehicle to create this connection.

Exercises such as $50 \div 6$ might be called "near facts." Division that do not come out evenly are much more prevalent in computations and in real situations than division facts or division without remainders. To determine the answer to $50 \div 6$, most people run through a short sequence of the multiplication facts, comparing each product to $50$: "6 times 7 (too low), 6 times 8 (too high), 6 times 9 (high). Must be 8. That's 48 and 2 left over." This process can and should be drilled. That is, children should be able to do problems with one-digit divisors and one-digit answers plus remainders mentally and with reasonable speed.

**Activity 3.10**

**How Close Can You Get?**

To practice "near facts," try this exercise. As illustrated, the idea is to find the one-digit factor that makes the product as close as possible to the target without going over. Help children develop the process of going through the multiplication facts that were just described. This can be a drill with the full class by preparing a list for the overhead, or it can be a worksheet activity.

- Find the largest factor without going over the target number.
  - $4 \times \boxed{6} \rightarrow 23$ (left over)
  - $7 \times \boxed{6} \rightarrow 52$ (left over)
  - $6 \times \boxed{9} \rightarrow 27$ (left over)
  - $9 \times \boxed{6} \rightarrow 60$ (left over)
Effective Drill

- There is little doubt that strategy development and general number sense (number relationships and operation meanings) are the best contributors to fact mastery. Drill in the absence of these factors has repeatedly been demonstrated as ineffective. However, the positive value of drill should not be completely ignored. Drill of nearly any mental activity strengthens memory and retrieval capabilities.

When and How to Drill

Teachers and parents hold tenaciously to their belief in drill. Undoubtedly, far too much time is devoted to inefficient drill of basic facts, often with a negative impact on students' attitudes toward mathematics and beliefs in their abilities.

Avoid Inefficient Drill

Adopt this simple rule and stick with it: Do not subject any student to fact drills unless the student has developed an efficient strategy for the facts included in the drill. Drill can strengthen strategies with which students feel comfortable—ones they “own”—and will help to make these strategies increasingly automatic. Therefore, drill of strategies such as those discussed in this chapter will allow students to use them with increased efficiency, even to the point of recalling the fact without being conscious of using a strategy. Counting on fingers and making marks on paper can never result in automatic fact recall regardless of the amount of drill. Drill without an efficient strategy present offers no assistance.

The preceding statement even applies to students in the upper elementary grades who have not yet mastered facts. Because the curriculum at these levels typically does not include strategy development, drill is often the only approach offered. Alternatives to this serious error are discussed later in the chapter.

Individualize Drill

It is unreasonable to expect every student in your class to develop and be comfortable with the same strategies. As you have seen, there are multiple paths to mastery. Different students will bring different number tools to the task and will develop strategies at different rates. This means that there are few drills that are likely to be efficient for a full class at any given time. That is why so many of the suggested activities are designed as flash cards, games, or simple repeatable worksheets. By creating a large number of drill activities promoting different strategies and addressing different collections of facts, it is not at all unreasonable to direct students to activities that are most useful for them.

It is important to listen carefully to the strategies different students are using. For example, if a student tends to solve multiplication facts with a 5 by multiplying by 10 and subtracting, it is not profitable to push the “nifty nine” strategy. A student who has not mastered addition facts is not ready for subtraction practice.

Drill for Strategy Retrieval

When a fact is presented without a reminder of a strategy, students need to select from their memory the mental method that works best for that fact. Drills can be devised that help students look at a fact and recall a strategy that works. The next activity suggests how this might be done.

ACTIVITY 3.11

Sort Them as You Do Them

Mix ordinary flash cards from two or more strategies into a single packet. Prepare simple pictures or labels for the strategies in the packet. Students first match a card with a strategy and then use the strategy to answer that fact.

"Sort Them as You Do Them" can be tailored to match the strategies that an individual student is using and working on. Talk with the student, and have him help you put the activity together.

Technology Note

With literally hundreds of software programs that offer drill of basic facts, it is easy to assume that programs offer games or exercises at various difficulty levels. Yet, there do not seem to be any programs that organize the strategies. It should be clear that computerized fact practice should not replace instruction. Instead, these programs should enhance the instruction. Use software as a tool to keep performance records of students' achievement. There are numerous sites on the Internet that offer drill and practice that are modifiable and many provide options for the size of the numbers and whether or not to time.

What About Timed Tests?

Consider the following:

Teachers who use timed tests believe that the tests help children learn basic facts. This makes no instructional sense. Children who perform well under time pressure display their skills. Children who have difficulty with skills, or who work more slowly, run the risk of reinforcing wrong learning under pressure. In addition, children can become fearful and negative toward their math learning. (Burns, 2000, p. 157)

Think about this quotation whenever you are tempted to give a timed test. Reasoning and pattern searching are never facilitated by restricting time. Some children simply cannot work well under pressure or in situations that provoke stress.

Although speed may encourage children to memorize facts, it is effective only for students who are goal oriented and who perform in pressure situations. The pressure of speed can be debilitating and provides no positive benefits.
The value of speed drills or timed tests as a learning tool can be summed up as follows:

**Timed tests**
- Cannot promote reasoned approaches to fact mastery
- Will produce few long-lasting results
- Reward few
- Punish many
- Should generally be avoided

**Assessment Note**

It is possible to have a purpose for a timed test of basic facts if it may be determined that combinations are mastered and can be applied to other situations. Even for diagnostic purposes there is little point in having more than one or two every couple of months.

**Fact Remediation with Upper-Grade Students**

- Students who have not mastered their basic facts by the fifth grade are in need of something other than more drill. They have certainly seen and practiced facts countless times in previous grades. There is no reason to believe that the drills you provide will somehow be more effective than last year's. These students need something better. The following key ideas can guide your efforts to help these older students.

1. **Recognize that more drill will not work.** Students' fact difficulties are due to a failure to develop or to connect concepts and relationships such as those that have been discussed in this chapter, not a lack of drill. At best, more drill will provide temporary results. At worst, it will cause negative attitudes about mathematics.

2. **Inventory the known and unknown facts for each student in need.** Find out from each student what facts are known quickly and comfortably and which are not. Fifth- and sixth-grade students can do this diagnosis for you. Provide sheets of all facts for one operation in random order and have the students circle the facts they are hesitant to state and answer all others. Suggest that finger counting or making marks in the margins is not permitted.

3. **Diagnose strengths and weaknesses.** Find out what students do when they encounter one of their unknown facts. Do they count on their fingers? Add up numbers in the margins? Guess? Try to use a related fact? Write down times tables? Are they able to use any of the relationships that might be helpful in this chapter? Some of this you may be able to accomplish by having students write about how they approach two or three specific facts. More efficiently, you should conduct a 15-minute diagnostic interview with each student in need. Simply pose unknown facts and ask the student how he or she approaches them. Try an idea from this chapter and see what connections are already there. Don't try to teach; just find out.

4. **Provide hope.** Children who have experienced difficulty with fact mastery often believe that they cannot learn facts or that they are doomed to finger counting forever. Let these children know that you will help them and that you will provide some new ideas that will help them as well. Take that burden on yourself and spare them the prospect of more defeat.

5. **Build in success.** As you begin a well-designed fact program for a child who has experienced failure, be sure that successes come quickly and easily. Begin with easy strategies, and introduce only a few new facts at a time. Even with pure drill, repetitive exposure to five facts in three days will provide more success than introducing 15 facts in a week. Success builds success! With strategies as an added assist, success comes even more quickly. Point out to children how one idea, one strategy, is all that is required to learn many facts. Use fact charts to show what set of facts you are working on. It is surprising how the chart quickly fills up with mastered facts. Keep reviewing newly learned facts and those that were already known. This is success. It feels good and failures are not as apparent. Short practice exercises can be designed as homework. Explain strategies and build them into the exercises. At the end of the exercises, have students write about which ideas are helpful and which are not. Use this information to design the next exercise.

Your extra effort beyond class time can be a motivation to a student to make some personal effort on his or her own time. During class, these students should continue to work with all students on the regular curriculum. You must believe and communicate to these students that the reason they have not mastered basic facts is not a reflection of their ability. With efficient strategies and individual effort, success will come. Belief is possible.

**Facts: No Barrier to Good Mathematics**

- Students who have total command of basic facts do not necessarily reason better than those who, for whatever reason, have not yet mastered facts. Today, mathematics is not about computation, especially pencil-and-paper computation. Mathematics is about reasoning and patterns and making sense of things. Mathematics is problem solving. There is no reason that a student who has not yet mastered all basic facts should be excluded from real mathematical experiences.

The most obvious alternative is the calculator. It should be on the desk every day for all students. There is absolutely no evidence that the presence of a calculator will impede basic fact mastery. On the contrary, the more students use the calculator, the more proficient they will be with it. This will make many of the calculator fact drills more effective and provide students with ready access to electronic flash cards. In a classroom climate where most students do know their facts and where students help one another and share thinking strategies as has been suggested, very few students will rely on the calculator during any prolonged period. In fact, when equipped with effective strategies, many students are using the calculator to complete basic facts as slowing them down.

Students who are relegated to drill of facts when the rest of the class is engaged in meaningful experiences will soon feel stupid and incapable of doing "real" mathematics. By contrast, when students who have not mastered facts are engaged in exciting and meaningful experiences, they have real motivation to learn facts and real opportunities to develop relationships that can aid in that endeavor. Do not allow students who are behind in fact mastery to fall behind in mathematics.
If You Didn't Know

Based on activity 1.4, p. 92

Grade Level: Third or fourth grade

Mathematics Goals:
1. To develop students' mental strategies for basic multiplication facts.
2. To develop problem-solving skills in the context of basic multiplication facts.

Thinking About the Students:
Students understand that multiplication is the same thing as repeated addition. They are proficient with their addition facts and have mastered the easier multiplication facts but still struggle with some of the more difficult multiplication facts.

Materials and Preparation:
- A stack of multiplication flash cards that most students are having difficulty mastering.
- Each student will have a copy of a ten-by-ten multiplication array (see Blackline Master 1.5) and a tabloid L (as shown in Figure 3.14) to outline specific product arrays.
- A transparency of the ten-by-ten dot array for the "After" portion of the lesson. You will also need a tabloid L to outline specific product arrays on the overhead.

Lesson

Before:

Ask students to remember when they were learning addition facts and how knowing 6 + 6 could help them figure out 6 + 7. Elicit student ideas. The point to emphasize is what is meant by a fact that they know (e.g., a fact that they have already mastered and know without counting).

Ask students to pretend that they do not know 6 x 5 but that they do know 5 x 5. Use the transparency of the ten-by-ten dot array and the tabloid L to illustrate a 6 x 5 array on the overhead. How could they use 5 x 5 to help them determine 6 x 5? Elicit student ideas. For example, 5 x 5 is 25 and one more 5 is 30.

Without using the dot array, ask students to suppose they know 3 x 3 but not 6 x 5. How could they use 3 x 5 to determine 6 x 5? Again elicit student ideas. For example, 3 x 5 is 15 and means 3 groups of 5; 6 x 5 means 6 groups of 5, so just double the 15 to get 30.

The Task:

If you did not know the answer to 6 x 8, how could you figure it out by using something that you do know?

Establish Expectations:

• Explain to students that their methods should be something that they can do in their heads and should not rely on counting. In other words, they should use a fact that they know—a fact that they have already mastered and know without counting.

• Encourage students to come up with more than one way.

• Explain that the ten-by-ten array is only one tool to help them think about different strategies. They do not have to use this tool.

During:

• If students have difficulty getting started, first ask them what 6 x 8 means in terms of addition (e.g., 6 groups of 8). Suggest they write the 8 eights either vertically or horizontally and look for different ways to group the numbers so that they can determine the answer more quickly. Do not press for a particular approach. Alter-

necssarily suggest that they group the 8's in twos and group the 16 pieces by four.

• Encourage students to come up with more than one pathway to find 6 x 8. You might have to suggest ideas to get them started.

• As students work, select a few to explain their strategies. Students need to present evidence in how they can better explain different strategies to their classmates.

After:

• Use a think-pair-share approach in which students discuss their ideas with a partner before they share them with the class.

• Ask students to share their methods of thinking about how to determine 6 x 8 mentally; they may need to accompany their explanations with drawings or equations so that classmates can follow along. You may need to step in to help students make explicit the particular strategy they are using, such as half then double, add one more set, and so on.

• Help students make connections between symbolic approaches such as a listing of 6 eights and a portion of the ten-by-ten array.

• Ideas students may use include 3 x 8 and then double; 4 x 6 and then double; 5 x 8 and one more 8, and double-double-double (12, 24, 48). Each of these ideas can be used with other hard multiplication facts as well. For a given strategy, challenge students to find other facts with which they can use the same approach.

Assessment Notes:

• Look for students who only use repeated addition to determine the answer to the multiplication fact. Have these students mastered the easier multiplication facts?

• Some students may need skills with mental addition to make a strategy useful.

For example, to use 3 x 8 and then double, a student must be able to double 24 mentally.

• Use word problems as a vehicle for prompting and developing different strategies. For example, to prompt the double and double again approach, pose a word problem such as the following: Mike and Sarah were making goody bags for the end-of-the-year party. They wanted to put 6 pieces of candy in each student's bag. Mike made 2 bags while Sarah made 4 bags. How many pieces of candy did each person use?

• For students having difficulty with easier facts, provide meaningful practice with strategies for groups of facts (double, fives, threes, nines) as described in this chapter.